

# REGULARIZATION OF ADJOINT ANALYSIS OF MULTISCALE CHAOTIC SYSTEMS

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Adjoint methods are widely used in many optimization, control and estimation problems encountered in fluid mechanics such as airfoil shape optimization, optimization of open-loop control distributions and variational data assimilation in numerical weather prediction (aka 4DVAR). When appropriately defined, the adjoint states allow one to conveniently express the gradient (i.e., the sensitivity) of a given cost functional to the control variables. The gradient thus obtained can then be used in an iterative optimization algorithm to find the optimal control distribution and the corresponding optimal states. The work [1] reviews the theory and discusses several applications.

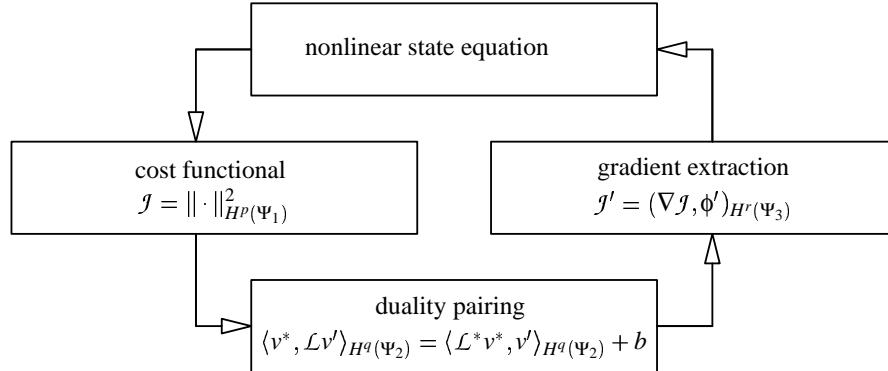


Figure 1: The four essential components of the adjoint-based optimization process.

Application of standard adjoint-based methods to genuinely multiscale and chaotic problems, such as high Reynolds number turbulence, often encounters significant difficulties. They include: presence of many local minima of the cost functional, non-smooth dependence of the problem solution on the data, and difficulties with numerical solution of the adjoint systems. Therefore, the general framework needs to be *regularized* and we propose here to do this by considering smoothness of the various fields involved in the adjoint analysis. We have identified four distinct places where such regularity can be introduced (see Fig. 1). First, one can regularize the evolution equation itself. Furthermore, in a control problem, there are three spatio-temporal domain of interest:  $\Phi_1$ , on which the cost function is defined,  $\Phi_2$ , on which the system evolution takes place, and  $\Phi_3$ , on which the control is defined. In an adjoint analysis, there is an inner product space associated with each of those domains, and  $L_2$  products are most commonly used. We propose here to derive the adjoint algorithm using more general Sobolev brackets  $H^p$ . In principle, the four regularization opportunities help us to (i) choose the evolution equation stressing a given aspect of the system dynamics, (ii) target the cost functional on the physics of interest, (iii) define numerically tractable adjoint operators, and (iv) extract suitably preconditioned gradients. We illustrate these regularization opportunities with computational examples of optimization problems for Kuramoto–Sivashinsky and Navier–Stokes systems.

## References

- [1] T. R. Bewley, “Flow control: new challenges for a new Renaissance”, *Progress in Aerospace Sciences* **37**, 21–58, (2001).